PMM U.S.S.R.,Vol.51,No.6,pp.808-810,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

THE PROBLEM OF THE STRUCTURE OF A DISCONTINUITY IN A STRAIN-HARDENING PLASTIC MEDIUM*

B.A. DRUYANOV and E.A. SVYATOVA

The problem of the structure of a velocity jump in an incompressible, strain-hardening viscoplastic material is considered, taking thermal effects into account. It is shown that a continuous solution of the problem does not always exist. At low viscosity a continuous solution exists only when the discontinuity v propagates at low velocities. It is established that when the viscosity tends to zero, the limiting solution may, under specific conditions, contain an isothermal velocity jump. In another limiting case, however, when the thermal conductivity tends to zero (adiabatic flow), a continuous solution exists for large values of v.

The problem of the structure of the discontinuity in a viscous, compressible, ideally plastic medium whose yield point depends on the mean pressure, taking thermal effects into account, was examined in /1/. A rigid-plastic strain-hardening material is investigated below. Certain specific formulations of the problem concerning the structure of the discontinuity were studied in /2, 3/. It will be apparent in what follows, that the problem is analogous, to some extent, to the problem of the structure of a discontinuity in a viscous heat conducting gas /4/.

Let us assume that the material is incompressible and viscoplastic. The yield point k, the thermal conductivity λ, the viscosity μ, and the internal energy per unit mass e are assumed to be known functions of the temperature θ and the Odqvist hardening parameter χ.

We denote the projections of the velocity on the Ox and Oy axes by u, v, respectively. Let us consider steady state flow of the medium experiencing pure shear, in the direction of the ordinate axis, assuming that $u = u(y)$, $v = \text{const}$. In this case the flow will be described by the following system of equations (a prime denotes a derivative with respect to y, ρ is the density of the medium):

$$\tau' = \rho v u', \ \rho v e' = (\lambda\theta')' + \tau u', \ \tau = k + \mu u', \ u' = v\chi' \tag{1}$$

(this is the equation of motion $(\tau = \tau_{xy})$, the equation of heat flux determining the ratio in the viscoplastic material, and the strain-hardening law).

Note that v is an essential parameter. The quantity having the opposite sign describes the velocity of propagation of the discontinuity. From (1) and the boundary conditions it follows that there is no solution when $v = 0$.

The boundary conditions are

$$u = \chi = \theta = \theta' = \chi' = 0, \ y \to -\infty \tag{2}$$

$$u = u_1, \ \theta' = \chi' = 0, \ y \to +\infty \tag{3}$$

Integrating Eqs.(1) and taking into account conditions (2), we obtain the following system of equations:

$$\rho v\chi' = M, \ M = \rho v^2\chi + k_0 - k \tag{4}$$

$$\lambda\theta' = L, \ L = \rho e - k_0\chi - {}^1/_2\rho v^2\chi^2 \ (k_0 = k(0, 0)) \tag{5}$$

Let us assume that $e(0, 0) = 0$. We eliminate the quantity u using the relation $u = v\chi$. From (4) and (5) we obtain

$$\lambda(\mu v^2)^{-1} d\theta/d\chi = L/M \tag{6}$$

The curve required must pass through the singularities of this equation corresponding to $y \to -\infty$ and $y \to +\infty$. We find the coordinates θ_1, χ_1 corresponding to $y \to \infty$ from the system of equations $L = 0, M = 0$ which determines, for the given value of u_1, θ_1, χ_1 and v. It is, however, more convenient to assume that the value of v is given and determine the quantities u_1, θ_1 and χ_1.

The singularity corresponding to $y \to -\infty$ has coordinates $\theta = \chi = 0$. The existence of the second singularity depends on the form of the curves $L = 0$, $M = 0$.

For the majority of practical materials we have $\partial e/\partial\theta, \partial e/\partial\chi, \partial k/\partial\chi > 0, \partial k/\partial\theta < 0$, and e depends weakly on χ /5/.

*Prikl.Matem.Mekhan.,51,6,1047-1049,1987

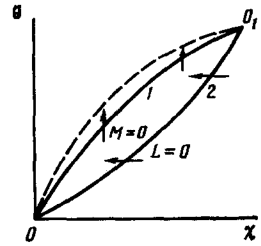

Fig.1

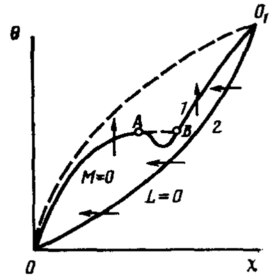

Fig.2

Let us consider the tangents of the angles of inclination of the curves $L=0$ and $M=0$ to the $O\chi$ axis:

$$\varkappa_1 = \left(k_0 + \rho v^2 - \rho\frac{\partial e}{\partial \chi}\right)\left(\rho\frac{\partial e}{\partial \theta}\right)^{-1}, \quad \varkappa_2 = \left(\frac{1}{\rho}\frac{\partial k}{\partial \chi} - v^2\right)\left(-\frac{1}{\rho}\frac{\partial k}{\partial \theta}\right)^{-1} \tag{7}$$

We see that the line $L=0$ represents a monotonically increasing concave curve (curve *2* in Fig.1 and 2). The form of the curve $M=0$ is determined by the relation $k(\theta, \chi)$.

For some metals the dependence of k on χ is represented by a convex curve. In this case the line $M=0$ has a maximum at the point $\rho v^2 = \partial k/\partial\chi$ (curve *1* in Fig.1). In the case of many other metals the strain-hardening curve has a concave segment. In this case the line $M=0$ will have two maxima and one minimum situated between them (curve *1* in Fig.2).

In all these cases the curves $L=0$ and $M=0$ have a second point of intersection situated in the quadrant $\theta>0, \chi>0$ if $\varkappa_2>\varkappa_1$, at the point $\theta=\chi=0$. In what follows, we shall assume that the above condition holds. Since $\varkappa_1(0,0)>0$, we must have at the point $\theta=\chi=0$ either $\varkappa_2>0$ or $\rho v^2<\partial k/\partial\chi$.

Let us bring in the velocity of propagation of weak discontinuities in a strain-hardening plastic (inviscid) medium $v_* = (\rho^{-1}\partial k/\partial\chi)^{1/2}$. The condition discussed above can be written in the form $v<v_0{}^*$.

Investigations show that under these conditions the point $\theta=\chi=0$ is a saddle point, and the point $\theta=\theta_1, \chi=\chi_1$ is a node. The solution therefore is a separatrix.

The reversible part of the energy accumulated at the microlevel and used up for strain hardening, is usually small compared with the specific work done by internal forces /5/. Therefore the condition that the dissipation should be positive can be written in the form $\tau u'>0$. This condition is satisfied when $u'>0$. Since $u'=v\chi'$, the solution must lie above the curve $M=0$, and this corresponds to the direction of internal curves indicated by arrows in Figs.1 and 2.

The curves corresponding to the solution are shown by the dashed lines.

We see from (6) that the solution can have an extremum only at the point where $L=0$. Under the assumptions made it will be a maximum. Consequently, the solution will increase monotonically in the interval $(0,\chi_1)$ and we must have $\varkappa_2>0$ at the point $O_1(\theta_1,\chi_1)$, i.e. $v<v_1{}^*$ where $v_1{}^*$ is the velocity of propagation of the weak discontinuities at the point $O_1(\theta_1,\chi_1)$. Thus, for a continuous solution to exist it is necessary that $v<v_1{}^*$. Usually we have $v_1{}^*<v_0{}^*$, therefore the condition $v<v_1{}^*$ contains within it the condition $v<v_0{}^*$.

When $\mu\to 0$, the solution tends to the curve $M=0$. If this curve has no extrema in the interval $(0,\chi_1)$, as is shown in Fig.1, then the solution is identical in the limit with $M=0$. If, on the other hand, there is an extremum (Fig.2), then the limit solution will contain an isothermal discontinuity in strain hardening and velocity (curve $OABO_1$ in Fig.2).

Although a solution exists when $\lambda\to 0$ (adiabatic flow), nevertheless under the conditions used, when the curve $L=0$ lies in the region $M<0$, it does not satisfy the condition that the energy dissipation must be positive. In this case the temperature is given in terms of the hardening parameter χ and becomes a parameter of the deformational softening.

Let us introduce the adiabatic yield point $k_a(\chi)=k_a(\theta,\chi)$. The function k_a is not necessarily monotonic, i.e. the material may harden of soften in various internals of variation of χ. Let k_a increase on some interval. In this case the equations will differ from the equations of isothermal flow only in the value of the yield point. As was shown in /2/, a continuous solution of the problem of the structure of the discontinuity exists, in the case of an isothermal flow, for large values of v, namely $v>(\rho^{-1}\partial k/\partial\chi)^{1/2}$ when $\chi=0$. The above argument is still valid in the case of an adiabatic flow of a strain hardening material when k_a is an increasing function, but in this case k must represent the adiabatic yield point k_a.

REFERENCES

1. PACHENSKII YA.A., On the structure of shock waves in elastoplastic media. PMM, 37, 2, 1973.
2. DRUYANOV B.A., Shear waves in strain hardening rigid plastic bodies. PMM, 44, 4, 1980.
3. DRUYANOV B.A. and SVYATOVA E.A., On strong velocity jumps in thermoplastic bodies. Dokl. Akad. Nauk SSSR, 262, 2, 1982.
4. ROZHDESTVENSKII B.L. and YANENKO N.N., Systems of Quasilinear Equations and Their Use in Gas Dynamics, Moscow, Nauka, 1968.
5. RANETSKII B. and SAVCHUK A., Temperature effects in plasticity. Pt.1, Related theory. Problems of the Theory of Plasticity and Creep. Moscow, Mir, 1979.

Translated by L.K.